LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **ALLIED**

FOURTH SEMESTER – **APRIL 2023**

MT 4205 – BUSINESS MATHEMATICS

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A

Answer ALL questions: **(10 x 2 = 20)**

1. Define cost function of a firm.
2. If the marginal function for output is given by $R_m = \frac{6}{(x+2)^2} + 5$, find the total revenue function by integration.
3. Find $\frac{dy}{dx}$ if $y = x^2 + y^2 - 2x$.
4. Identify the elasticity of the function $x = \frac{27}{p^3}$.
5. Find the nth derivative for the function $y = e^{ax}$.
6. Evaluate $\int (8x^7 - 5x^4 - 1)dx$.
7. If $A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}$, find $2A$.
8. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$, then find $A + B$.
9. Given $\frac{x+1}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$ then find A and B.
10. Define solution in linear programming problem.

SECTION B

Answer ANY FIVE questions: **(5 x 8 = 40)**

11. The total cost C for output x is given by $C = \frac{2}{3}x + \frac{35}{2}$. Find the cost when output is 4 units, also find the average cost of 10 units.
12. For the following pair of demand functions for two commodities X_1 and X_2, determine the four partial marginal demands, the nature of relationship (Complementary, Competitive or neither) between X_1 and X_2 and the four partial elasticities of demand $x_1 = \frac{4}{p_1{}^2 p_2}$ and $x_2 = \frac{16}{p_2{}^2 p_1}$.
13. If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.
14. Find the maximum and minimum values of the function $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$.
15. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$.
16. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.
17. Compute the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{pmatrix}$.
18. Solve the equations $2x - 3y = 3$, $4x - y = 11$ using matrix method.

PART C

Answer any TWO questions: **(2 X 20 = 40)**

19. (a) If AR and MR denote the average and marginal revenue at any output, show that elasticity of demand is equal to $\frac{AR}{AR-MR}$. Verify this for the linear demand law $p = a + bx$.

    (b) If the marginal revenue function for output x is given by $R_m = \frac{6}{(x+2)^2} + 5$, find the total revenue by integration. Also deduce the demand function. (10+10)

20. (a) If $u = x^2y^3z^4$. Find $\frac{\partial u}{\partial x}$, $\partial u/\partial y$, $\partial u/\partial z$.

    (b) Integrate x^2e^x with respect to x. (10+10)

21. Evaluate $\int \frac{(3x+7)}{2x^2+3x-2}dx$.

22. (a) Solve by Cramer's rule $2x+y-z=3$; $x+y+z=1$; $x-2y-3z=4$.

    (b) A factory manufactures two articles A and B. To manufacture the article A, a certain machine has to be worked for 1.5 hours and in addition a craftsman has to work for 2 hours. To manufacture the article B, the machine has for 1.5 hours. In a week the factory can avail of 80 hours of machine time and 70 hours craftsman's time. The profit on each article A is Rs. 5 and that on each article B is Rs. 4. If all the articles produced can be sold away, how many of each kind should produce to earn the maximum profit per week. Formulate the linear programming problem. (10+10)
